

# INTERNATIONAL STANDARD

# NORME INTERNATIONALE

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**Mathematical expressions for reliability, availability, maintainability and maintenance support terms**

**Expressions mathématiques pour les termes de fiabilité, de disponibilité, de maintenabilité et de logistique de maintenance**



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INTERNATIONAL  
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# INTERNATIONAL ELECTROTECHNICAL COMMISSION

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## **MATHEMATICAL EXPRESSIONS FOR RELIABILITY, AVAILABILITY, MAINTAINABILITY AND MAINTENANCE SUPPORT TERMS**

### FOREWORD

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International Standard IEC 61703 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition published in 2001. This edition constitutes a technical revision.

This edition includes the following significant technical changes with respect to the previous edition:

- a) standard made as self containing as possible;
- b) item split between individual items and systems;
- c) generalization of the dependability concepts for systems made of several components;
  - introduction of the conditional failure intensity (Vesely failure rate);
  - introduction of the state-transition and the Markovian models;

- generalization of the availability to production availability;
- d) introduction of curves to illustrate the various concepts.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/1682/FDIS	56/1693/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

This International Standard is to be used in conjunction with IEC 60050-192:2015.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC website under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

## INTRODUCTION

IEC 60050-192 provides definitions for dependability and its influencing factors, reliability, availability, maintainability and maintenance support, together with definitions of other related terms commonly used in this field. Some of these terms are measures of specific dependability characteristics, which can be expressed mathematically.

It is important for the users to understand the mathematical meaning of those expressions and how they are established. This is the purpose of the present International Standard which, used in conjunction with IEC 60050-192, provides practical guidance essential for the quantification of those measures. For those requiring further information, for example on detailed statistical methods, reference should be made to the IEC 60605 series [23]<sup>1</sup>.

Annex A provides a diagrammatic explanation of the relationships between some basic dependability terms, related random variables, probabilistic descriptors and modifiers.

Annex B provides a summary of measures related to time to failure.

Annex C compares some dependability measures for continuously operating items.

The bibliography gives references for the mathematical basis of this standard, in particular, the mathematical material is based on references [2], [6], [8], [9], [13], [14] and [18]; the renewal theory (renewal and alternating renewal processes) can be found in [6], [8], [9], [10], [11], [13], [15], and [17]; and more advanced treatment of renewal theory can be found in references [1], [3], [12], [16], [19] and [20]. More information on the theory and applications of Markov processes can be found in references [3], [9], [10], [15], [16], [17] and [19].

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<sup>1</sup> Numbers in brackets refer to the Bibliography.

# MATHEMATICAL EXPRESSIONS FOR RELIABILITY, AVAILABILITY, MAINTAINABILITY AND MAINTENANCE SUPPORT TERMS

## 1 Scope

This International Standard provides mathematical expressions for selected reliability, availability, maintainability and maintenance support measures defined in IEC 60050-192:2015. In addition, it introduces some terms not covered in IEC 60050-192:2015. They are related to aspects of the system of item classes (see hereafter).

According to IEC 60050-192:2015, dependability [192-01-22] is the ability of an item to perform as and when required and an item [192-01-01] can be an individual part, component, device, functional unit, equipment, subsystem, or system.

To account for mathematical constraints, this standard splits the items between the individual items considered as a whole (e.g. individual components) and the systems made of several individual items. It provides general considerations for the mathematical expressions for systems as well as individual items but the individual items which are easier to model are analysed in more detail with regards to their repair aspects.

The following item classes are considered separately:

- Systems;
- Individual items:
  - non-repairable [192-01-12];
  - repairable [192-01-11]:
    - i) with zero (or negligible) time to restoration;
    - ii) with non-zero time to restoration.

In order to explain the dependability concepts which can be difficult to understand, keep the standard self-contained and the mathematical formulae as simple as possible, the following basic mathematical models are used in this standard to quantify dependability measures:

- Systems:
  - state-transition models;
  - Markovian models.
- Individual items:
  - random variable (time to failure) for non-repairable items;
  - simple (ordinary) renewal process for repairable items with zero time to restoration;
  - simple (ordinary) alternating renewal process for repairable items with non-zero time to restoration.

The application of each dependability measure is illustrated by means of simple examples.

This standard is mainly applicable to hardware dependability, but many terms and their definitions may be applied to items containing software.